Magnetic Wall From Chiral Phase Transition and CMBR Correlations

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Possible CMBR correlations are estimated for a model in which a Hubble-size magnetic wall is formed during the QCD chiral phase transition. Measureable polarization correlations are found for l-values greater than about 1000. It is also found that metric perturbations from the wall could give rise to observable CMBR correlations for large l.

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I. INTRODUCTION

In the present paper the possibility is explored that large-scale magnetic structures created during the QCD (Quantum Chromodynamic) chiral phase transition might lead to observable CMBR (cosmic microwave background radiation) correlations . This work is motivated by improved CMBR observations in progress, which promise polarization and temperature correlation measurements with multipoles l in the thousands. These magnetic structures could also be primoidal seeds of galactic and extra-galactic magnetic fields. A long-standing problem of astrophysics is the origin of the large-scale galactic and extra-galactic magnetic fields which have been observed. Of particular importance for cosmology is the possible seeding of these magnetic structures by primoidal, early-universe, magnetic structures. For a recent review see Ref [1]. We do not, however, investigate galactic or extra-galactic magnetic structure seeded by the QCD phase transition in the present work, but center on CMBR polarization and gravitational wave correlations,

In most of the theoretical treatments, including inflationary models, the magnetic fields arise from electrically charged particle motion. Considerations of nucleosynthesis and CMBR and the galactic magnetic fields have been used to constrain the magnitudes of the primoidal tangled (random) magnetic fields to values of about 10^{-9} Gauss, although in a recent model [2] fields of two orders of magnitude larger are consistent with CMBR observations. Constraints on homogeneous primoidal magnetic fields from CMBR have also been determined [3].

Early universe phase transitions are of great interest. The electroweak and QCD chiral phase transitions are of particular interest as common ground for particle physics and astrophysics. For the electroweak phase transition, using an Abelian Higgs model [4] with the QED lagrangian included, magnetic field generation has been calculated [5,6] in bubble collisions. For the QCD phase transition primoidal magnetic fields generated from charged currents at the bubble surfaces during the nucleation have been estimated [7,8] to be as large as 10^8 Gauss, and still be consistent with observed values of galactic and extra-galactic fields. There has also been a recent calculation [9] of the CMBR power spectrum from density perturbations caused by promoidal magnetic fields that is similar to the calculation of Ref [2], but for scalar perturbations.

The present paper is motivated by our QCD instanton model of bubble walls formed during the QCD chiral phase transition [10] and by the recent work of Forbes and Zhitnitsky, who have used a QCD domain wall [11] as a mechanism for generating magnetic fields which could evolve to large-scale galactic fields [12]. Considering classical theory of bubble collisions, in which walls with the same surface tension as the colliding bubbles are formed within the merged bubbles, it was conjectured [10] that hubble-scale instanton walls might be formed, with lifetimes sufficient to form magnetic walls. In section II the description of the QCD phase transition in our instanton model and the possible resulting magnetic wall are discussed. It is shown that with an interior instanton wall, which is similar to QCD domain walls, the magnetic wall formed in the hadronic phase is similar to that modelled in Ref [12]. In section III the evolution of the magnetic structure to the time of recombination, and the CMBR polarization correlations are derived. In section IV the correlations from metric perturbations are derived, and in section V we give our conclusions.

II. QCD CHIRAL PHASE TRANSITION AND MAGNETIC WALL

In our model [10] of the bubbles of the hadronic-phase universe nucleating within the quark-gluon phase universe during the QCD chiral phase transition, which we assume is first order, we use the purely gluonic QCD Lagrangian density, $\mathcal{L}^{glue} = G^a_{\mu\nu}G^{\mu\nu a}/4$. $G^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig[A_\mu, A_\nu]$ is the color field tensor, defined in terms of the gluonic color field $A_\mu = A^n_\mu \lambda^n/2$, where λ^n are the eight SU(3) Gell-Mann matrices, with the notation that (μ,ν)= (1...4) and (a,b,...) = (1,2,3) are Dirac indices and color indices, respectively. Recognizing that it has been shown that QCD instantons can represent the midrange nonperturbative aspects of QCD, we use the instanton model [13] for the color field

$$A^{n,inst}_\mu(x) = \frac{2\eta^{-n}_{\mu\nu}x^\nu}{(x^2+\rho^2)} \tag{1}$$

$$G^{n,inst}_{\mu\nu}(x) = -\frac{\eta^{-n}_{\mu\nu}4\rho^2}{(x^2+\rho^2)^2}, \tag{2}$$

for the instanton and a similar expression with -n for the anti-instanton, where ρ is the instanton size and the $\eta^n_{\mu\nu}$ are defined as [13]

$$\begin{aligned} \eta^a_{ij} &= \epsilon_{aij} \\ \eta^a_{\mu 4} &= \delta_{a\mu} \\ \eta^a_{4\nu} &= -\delta_{a\nu}, \end{aligned} \tag{3}$$

with (i,j) = (1,2,3) and a, μ, ν as defined above. The QCD instanton model, reviewed in Ref [14], has been a very useful representation of nonperturbative QCD for hadronic properties. For example, one can successfully predict the known scalar glueball candidates using this formalism [15].

Noting that the wall of the bubble separating the quark-gluon phase from the hadronic phase is gluonic in nature, and that the instanton model is successful in representating midrange nonperturative QCD, we have recently formulated [10] an instanton model of the bubble wall. The starting point is the energy-momentum tensor of pure gluonic QCD:

$$T^{\mu\nu} = G^{\mu\alpha}_a G^\nu_{\alpha a} - \frac{1}{4}g^{\mu\nu}G^{\alpha\beta}_a G_{\alpha\beta a}, \tag{4}$$

which gives in the instanton model at finite temperature in Minkowski space the spatial energy momentum tensor

$$T^{ij,inst} = \left(\frac{4\overline{\rho}^2\overline{N}}{(x^2+\overline{\rho}^2)^2}\right)^2 \delta_{ij}, \tag{5}$$

and the energy density,

$$T^{00,inst} = 96\left(\frac{\overline{\rho}^2\overline{N}}{(x^2+\overline{\rho}^2)^2}\right)^2, \tag{6}$$

where $\overline{\rho}$ is the instanton size and $\overline{N}$ is the instanton density at the bubble surface at temperature T=T$_c$, the temperature of the chiral phase transition, approximately 150 MeV. $\overline{N}$ is determined from the tunneling amplitude [13,14], and $\overline{\rho}$ has been found in finite temperature calculations [16,17] to be $\overline{\rho} \simeq 0.25 fm$. In Ref [10] the energy density, T^{00} was shown to be consistent with numerical calculations of the surface tension. T^{ij} can be used to calculate bubble collisions.

Recently, effective field models have been used to calculate QCD domain walls which could form within bubbles during the QCD chiral phase transition [11]. These domain walls have a space-time structure very similar to a wall composed of instantons, with the form given in Eq.(1). Such a wall could interact with nucleons in the hadronic phase to produce electromagnetic structures via effective magnetic and electric dipole moments [18] of the nucleon field. This model was used [12] to investigate possible primoidal magnetic fields at the time of the chiral phase transition that could lead to large-scale galactic magnetic fields.

As pointed out in Ref. [10], the collision of nucleating bubbles during the phase transition could lead to an interior gluonic wall. If the theory of classical bubble collisions can applied, the interior wall would be similar to the bubble walls, with the same surface tension. In other words, there would be an instanton wall with an energy-momentum tensor given by Eq.(5,6). In a 1+1 model of colliding bubble walls based on QCD we recently found [19] that, with instanton-like boundary conditions, an instanton-like internal wall does seem to form in the collision region. Although this model is too simple for studying the nucleation problem, there have been a number of investigations of the QCD chiral phase transition using classical nucleation theories with effective Lagrangians [20–23]. These studies find that in contrast to the electroweak phase transition where many bubles nucleate, collide and merge, the QCD chiral phase transition seems to proceed via inhomogeneous nucleation, with larger distance between bubbles and rather few nucleating bubbles involved in the transition to the hadronic phase. Therefore, it is likely that very few large-scale instanton walls were formed, and that the calculation of CMBR correlations from one magnetic wall, which is the picture used in the present study, is a good starting point.

Recognizing that the mathematical form of the instanton and domain walls are very similar, the arguments of Ref. [12], including estimates of the lifetime of the interior QCD instanton wall, can be applied to estimate the primary electromagnetic wall that might have been formed at t$\simeq 10^{-4}$ sec. The magnetic wall is formed by the interaction of the nucleons with the gluonic wall, with the electromagnetic interaction Lagrangian

$$\mathcal{L}^{int} = -e\bar{\Psi}\gamma^{\mu}A^{em}_{\mu}\Psi, \tag{7}$$

where Ψ is the nucleon field operator. This leads to the electromagnetic interaction with the nucleons magnetic dipole moment given in terms of the electromagnetic field tensor, $F^{\mu\nu}$ by

$$\mathcal{V}^{int} = \frac{e}{2M_n}\bar{\Psi}\sigma_{\mu\nu}\gamma_5\Psi F^{\mu\nu}, \tag{8}$$

and a similar interaction (without the γ_5) for the electric dipole moment, due to cp violation. From Eq.(8) one can estimate the magnetic field in the wall. In analogy to the classical theory in which the magnetic field within a magnetized object is given by the magnetic dipole moment density [24] within a factor depending on the shape of the object, the B-field is given by the matrix element of $\bar{\Psi}\sigma_{\mu\nu}\gamma_5\Psi$. For the gluonic instanton wall oriented in the x-y direction one obtains for $B_z \equiv B_W = F^{21}$ within the wall of thickness ρ

$$B_z \simeq \frac{1}{\rho\Lambda_{QCD}}\frac{e}{2M_n} < \bar{\Psi}\sigma_{21}\gamma_5\Psi > . \tag{9}$$

A suppression factor of $(\rho\Lambda_{QCD})^{-1}$ has been used in Eq.(9) since alligned dipoles tend to cancel, as discussed at length in Ref [12]. The matrix element in Eq.(9) is estimated using the Fermi momentum in the plane of the wall during the QCD phase transition as Λ_{QCD}, giving $< \bar{\Psi}\sigma_{21}\gamma_5\Psi >= \frac{4\pi}{(2\pi)^2}\Lambda^2_{QCD}$, with a factor of four from the spin and isospin degeneracy. Using Λ_{QCD} = 150 Mev, the resulting magnitude of the magnetic field at the wall is

$$B_W \simeq \frac{3e}{14\pi}\Lambda_{QCD}, \tag{10}$$

which is essentially the same as the estimate of Ref [12], within the errors of the scale factors. The calculation of the electric field is similar, giving $E_z \simeq B_z \simeq B_W \simeq 10^{17}$ Gauss (within the wall).

Therefore our picture is that at the end of the QCD chiral phase transition there is a magnetic wall in the hadronic phase, which we model as

$$\mathbf{B}_W(\mathbf{x}) = B_W e^{-b^2(x^2+y^2)}e^{-M_n^2 z^2}, \tag{11}$$

or in momentum space

$$\mathbf{B}_W(\mathbf{k}) = \frac{B_W}{2\sqrt{2}b^2 M_n} e^{-(k_x^2+k_y^2)/4b^2} e^{-k_z^2/4M_n^2}, \tag{12}$$

where b^{-1} is of the scale of the horizon size, d_H, at the end of the chiral phase transition (t $\simeq 10^{-4}$ s), $b^{-1} = d_H \simeq$ few km, while $M_n^{-1} \simeq 0.2fm$. Therefore, although B_W is very large, since the wall occupies a very small volume of the universe, such a structure is compatible with nucleosynthesis, galaxy structure and the present CMBR observations.

In the present work we conjecture that the QCD bubble collisions lead to a magnetic wall given by Eq.(11) at 10^{-4}s and study the effects on CMBR polarization correlations and metric fluctuations.

III. CMBR POLARIZATION CORRELATIONS

In this section we investigate the polarization correlations arising from the electric and magnetic fields given in Eq.(11). The primoidal magnetic wall of the present work is quite different from the tangled fields considered previously [25,26], however, the evolution to the last scattering surface has many features in common with these studies. In addition to the polarization anisitropy which results from the magnetic wall itself, which gives rise to B-type polarization anisitropies discussed in the next subsection, in the subsequent subsection we also show that E-type anisotropies arise from scattering of the background radiation from the nucleon moments at the time of the phase transition, and show that the resulting power spectrum, C_l^{EE}, is too small to be detected.

A. Polarization Correlations From Magnetic Wall

In treating the temperature and polarization matrix for the Stokes parameters [27,28] we use the angular representation of Ref. [29] to derive the power spectrum of B-type polarization anisitropies, C_l^{BB}, which arise to a good approximation from the polarization source $P^{(1)} = -E_2^{(1)}/\sqrt{6}$, given by

$$E_2^{(1)} = \frac{1}{4}\sqrt{\frac{5}{6\pi}} \int d\Omega E(\hat{n}, x, \eta)(Y_2^1 - \sqrt{5}Y_1^1), \tag{13}$$

where the electromagnetic wave is propagating with $\mathbf{k} = k\hat{n}$, η is conformal time, and the Y_l^m are the standard spherical harmonics. From our model the source at the wall is $E_2^{(1)} = 5B_W/12\sqrt{2}b^2 M_n \equiv \mathcal{B}_W$ at the wall. The solutions of the Bolzman equation give the quantities B_l^m [29] at the time of recombination needed for the C_l^{BB}. To get the power spectrum we must evaluate $< B_z(\mathbf{k},\eta) B_z(\mathbf{k}',\eta) >$. Using the fact that $exp(-k^2 d_H^2) \simeq 1.0$ at the time of the chiral phase transition,

$$\begin{aligned} < B_z(\mathbf{k},\eta) B_z(\mathbf{k}',\eta) > &\simeq \mathcal{B}_W^2 \delta(k_x - k_x')\delta(k_y - k_y') < e^{-k_z^2/4M_n^2} e^{-k_z'^2/4M_n^2} > \\ &\simeq \mathcal{B}_W^2 d_H e^{-k_z^2/4M_n^2} \delta(\mathbf{k} - \mathbf{k}'). \end{aligned} \tag{14}$$

This gives for the polarization power spectrum

$$C_l^{BB} = \frac{(l+1)(l+2)}{\pi} \mathcal{B}_W^2 d_H \int dk k^2 \frac{j_l^2[k(\Delta\eta)]}{k^2(\Delta\eta)^2}, \tag{15}$$

where the conformal time integral over the visibility function has been carried out and $\Delta\eta$ is the conformal time width at the last scattering. The integral over the spherical Bessel function is carried out by using the fact that $j_l(x)$ peaks at l and that the integral $\int dz j_l^2(z) = \pi/(4l)$ for large l. Therefore in the range $100 < l < 2000$ we have the approximate result

$$C_l^{BB} \simeq \frac{25 d_H^5 B_W^2}{1152 M_n^2 \Delta\eta^3} l^2. \tag{16}$$

Using the parameters $M_n \Delta\eta = 1.5 \times 10^{39}$ (from Refs. [26,30]), $d_H = 0.37 \times 10^{24} GeV^{-1}$, and $B_W = 1.0 \times 10^{17}$ Gauss,

$$C_l^{BB} \simeq 4.25 \times 10^{-8} l^2 \tag{17}$$

The result for the B-type power spectrum is shown in Fig. 1 by the solid line. There have been a number of investigations of the polarization predicted by inflationary models [31] which show that the B-type polarization in inflationary models is smaller than the E-type, and that $l(l+1)C_l^{BB}$ peaks at l-values about 100. Ref [32] reviews, with extensive references, predictions of inflationary models and effects resulting from cosmological phase transitions. In Fig. 1 results from Ref [33] are shown. The curve shown by small circles is a string model normalized at $l = 100$ to our magnetic wall value, and with the same normalization the dashed curve gives a typical inflationary model result. Even the string model of Ref. [33] is two to three orders of magnitude smaller than the T-spectrum at the peak, One can see that for $l \simeq 1000$ the values of C_l^{BB} predicted by our magnetic wall picture exceed those of inflationary and topological models, and the l dependence is quite different.

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B. Polarization Correlations From Magnetic Dipole Scattering

In this subsection we discuss another possible source of polarization correlation arising from the horizon-size instanton wall created during the chiral phase transition and leading to an orientation of nucleon magnetic dipole moments. As the nucleon moments are alligned along the wall the background radiation will scatter from these moments producing scattered waves and magnetic structure which would give polarization at the last scattering surface. Taking the thickness of the instanton wall as the scale for the volume associated with the magnetic dipole moments, The density of moments, n, is obtained from the strength of the magnetic field:

$$B_z = \frac{2n\mu_n}{r_W^3}, \tag{18}$$

where μ_n is the neutron magnetic dipole moment and r_W is the wall thickness. Making use of the torque of the magnetic moment in a magnetic field $\mathbf{B}$, $d\mathbf{m}/dt = \mu_n^2 \mathbf{s} \times \mathbf{B}$, with $\mathbf{s}$ the spin of the neutron, one obtains the magnitude of the scattered B-field at time t=10^{-4} s: $B_{scat} = n^2\mu^2/2cos(\theta_s)kE_0 exp(ikr)/r$, with θ_s the angle between the neutron spin and the direction of the incident radiation and E_0 the E-field of the incident radiation. From this one obtains the Stokes parameter U, and in the notation of Ref [29] the polarization source

$$B_2^0(\mathbf{k}, \eta) = \frac{k^2 E_0^2}{2\sqrt{6}} (n\mu_n^2)^2. \tag{19}$$

Note that $E_2^0(\mathbf{k}, \eta) = B_2^0(\mathbf{k}, \eta)$, and results in E-type polarization anisotropies, C_l^{EE}.

Without a detailed calculation one sees that the resulting polarization correlations will be very small, since the extra k^2 dependence from the scattering from the moments introduces a factor of $(l^2/\Delta\eta)^2$. Therefore the C_l^{EE} polarization anisotropies resulting from the scattering from the nucleon moments at the time of the phase transition are too small to be measured.

IV. METRIC PERTURBATIONS FROM MAGNETIC WALL AND CMBR POWER SPECTRUM

In this section we derive the power spectrum from the gravitational waves arising from the magnetic wall of Sec.2. The calculation is very similar to that of Ref. [2], in which the power spectra was derived for tangled magnetic fields such as those considered in Refs. [25,26], with various scenarios for the scale dependence. Although our model of the narrow hubble-size magnetic structure is quite different, we can make use of much of the formalism of Ref. [2].

The stress-energy tensor [24] for our model magnetic wall, which has only the 33 spacial component, is

$$T_{33}(\mathbf{k}) = \frac{1}{8\pi}\int d^3q B_3(\mathbf{q})B_3(\mathbf{k}-\mathbf{q}) \qquad (20)$$
$$= \frac{2\pi^3\sqrt{\pi}}{M_n}B_W^2 e^{-\frac{3k_3^2}{8M_n^2}}$$

at the time of the chiral phase transition. Note that we are assuming that the wall is of hubble size in the x-y directions and have omitted the parts of the expression for the k_x, k_y dependence shown in Eq.(12). From Eq.(20) we obtain the B-wall power spectrum

$$< T_{33}(\mathbf{k},\eta)T_{33}(\mathbf{k}',\eta) > \; = \int d^3q d^3q' < B_3(\mathbf{q})B_3(\mathbf{k}-\mathbf{q})B_3(\mathbf{q}')B_3(\mathbf{q})B_3(\mathbf{k}'-\mathbf{q}') > . \qquad (21)$$

In evaluating Eq.(21) we use

$$< e^{-\frac{k^2}{8M_n^2}} e^{-\frac{k'^2}{8M_n^2}} \delta(k_x)\delta(k'_x)\delta(k_y)\delta(k'_y) = e^{-\frac{k'^2}{4M_n^2}} d_H \delta(\mathbf{k}-\mathbf{k}')). \qquad (22)$$

Using the notation of Ref [2] with a tensor projection,

$$< T_{33}(\mathbf{k},\eta)T_{33}(\mathbf{k}',\eta) > \; = \frac{4}{a^8} f^2(k^2)\delta(\mathbf{k}-\mathbf{k}')), \qquad (23)$$

with

$$f^2(k^2) = \frac{2^3\pi^9}{M_n^2} d_H B_W^4. \qquad (24)$$

With h_{ij} the tensorial perturbations of the Friedman universe, $ds^2 = a^2[-d\eta^2 + (\delta_{ij} + 2h_{ij})dx^i dx^j]$. The Einstein equations, using the representation $h_{ij} = 2HQ_{ij}^{(2)}$ of Ref [29], with $\nabla^2 Q_{ij}^{(2)} = -k^2 Q_{ij}^{(2)}$ are

$$\ddot{H} + 2\frac{\dot{a}}{a}\dot{H} + k^2 H = 8\pi G\frac{4}{a^8} f^2(k^2). \qquad (25)$$

The power spectrum for the tensor metric fluctuations are given by

$$< \dot{h}_{ij}(\mathbf{k}',\eta)\dot{h}_{ij}(\mathbf{k},\eta) > \; = 4|\dot{H}(\mathbf{k}))|^2 \delta(\mathbf{k}-\mathbf{k}')). \qquad (26)$$

Since Eq(25) was investigated in Ref [2], except with a different magnetic stress tensor, we use the solutions that they obtained. Assuming that the magnetic wall is formed at t=10^{-4} s in the radiation dominated epoch, at redshift z_{in}, and neglecting perturbations created after the time of matter-radiation equilibration η_{eq}, for $\eta > \eta_{eq}$ the approximate solution for $\dot{H}$ is

$$\dot{H}(k,\eta) = 4\pi G\eta_o^2 z_{eq} ln(\frac{z_{in}}{z_{eq}})\frac{j_2(k\eta)}{\eta} f(k). \qquad (27)$$

Carrying out integrals over Bessel functions, the solution for the metric fluctuation power spectrum is for $l >> 1$

$$C_l = \left[\frac{14}{25} G z_{eq} ln \frac{z_{in}}{z_{eq}}\right]^2 l^5 \eta_o^2 \int dz \frac{1}{z^4} f^2(z/\eta_o) J_{l+3}(z). \tag{28}$$

Using the form of f(z) given in Eq(24), the integral in Eq(28) is approximately

$$\int dz \frac{1}{z^4} f^2(z/\eta_o) J_{l+3}(z) \simeq \frac{8\pi^9 d_H}{M_n^2} B_W^4 \frac{0.106}{l^4} \tag{29}$$

for $M_n \eta_o >> l >> 1$. Taking t_{in} at 10^{-4} s or $z_{in} = 1.17 \times 10^8 z_{eq}$ and $d_H = 0.371 \times 10^{24} GeV^{-1}$, the power spectrum is ($l >> 1$)

$$l(l+1)C_l \simeq 6.9 \times 10^{-7} l^3 \tag{30}$$

for $B_W = 10^{17}$ Gauss. Therefore, the metric perturbations from the QCD-induced wall result in CMBR effects large compared to other tensor perturbations that have been estimated for for l values of the order of 1000.

V. CONCLUSIONS

We conlcude that if the QCD chiral phase transition produces a gluonic wall of domain size and nucleon thickness at the time of the final collision of nucleating bubbles, and produces a magnetic wall as that found in the domain wall model of Refs [11,12], it would result in polarization correlations which have a l-dependence for $l \simeq 1000$ different than other cosmological predictions and which should be measurable with the next generation of CMBR measurements. Also, there would be distinguishable temperature correlations arising from metric fluctuations. In order to investigate the collisions during nucleation to determine the details of the interior wall it is necessary to include quark and hadronic degrees of freedom to obtain the difference in the free energy in the two phases, a subject for future research.

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